SHENTON COLLEGE

Year 11 Mathematics Methods AEMAM Term 1 2020

Test 1 Counting and Probability

Calculator Free *Formula Sheet Allowed*

Student Name: Solutions

Teacher: Alfonsi Feutrill Loh McRae White

Time Allowed: 25 minutes

Calculator Free:	/33

*Attempt **all** questions.*
*All necessary working and reasoning must be shown for **full marks**.*
Marks may not be awarded for untidy or poorly arranged work.

Question 1. **[1, 1, 1, 1 = 4 marks]**

Consider two sets $A = \{1, 2, 3, 5, 8, 13, 21\}$ and $B = \{1, 3, 6, 10, 15, 21\}$ and the universal set $U = \{x : x \in \mathbb{N}, 1 \le x \le 22\}$, where $\mathbb{N}$ denotes the natural, or counting, numbers.

a) Use set notation to list the elements of the set:

(i) $A \cap B$

$= \{1, 3, 21\}$ ✓ lists all, without repetition

(ii) $A \cup B$

$= \{1, 2, 3, 5, 6, 8, 10, 13, 15, 21\}$ ✓ lists all

−1 notation

(iii) C, where $C = \{x : x \in A, \text{ x is even}\}$

$= \{2, 8\}$ ✓ lists all

b) State $n(\overline{A \cup B})$

$= 12$ ✓ states 12 allow $n(12)$

Either $n(A \cup B)' = n(U) - n(A \cap B)$
$= 22 - 10$
$= 12$

or $\{4, 7, 9, 11, 12, 14, 16, 17, 18, 19, 20, 22\}$

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Question 2. **[1, 2, 2, 2 = 7 marks]**

Evaluate

a) $10! \div 7!$

$= \frac{10!}{7!}$

$= 10 \times 9 \times 8$

$= \underline{\underline{720}}$ ✓ evaluates

b) $4! - 3! + 2! - 1! + 0!$

$= 24 - 6 + 2 - 1 + 1$ ✓ evaluates 0!

$= \underline{\underline{20}}$ ✓ evaluates

=19 (1 mark)
=15 (0 marks)

c) $\binom{8}{4} = \frac{8!}{(8-4)!\,4!}$ or $\frac{8!}{4!\,4!}$

✓ shows $\frac{n!}{(n-r)!\,r!}$ $= \frac{\cancel{8} \times 7 \times \cancel{6}^{2} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$

just evaluates ✓✓

$= \underline{\underline{70}}$ ✓ evaluates

d) ${}^{100}C_{99} + {}^{50}C_{50}$

$= 100 + 1$ ✓ ✓

evaluates ${}^{100}C_{99}$
evaluates ${}^{50}C_{50}$

$= \underline{\underline{101}}$

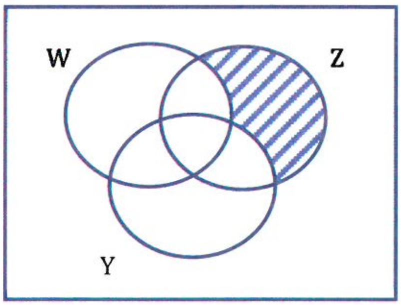
−1 if incorrect final answer

Question 3 **[1, 1 = 2 marks]**

a) Express the following shaded region using set notation

$Z \cap (\overline{W \cup Y})$

or $Z \cap \overline{W} \cap \overline{Y}$



$W' \cup \overline{Y}$

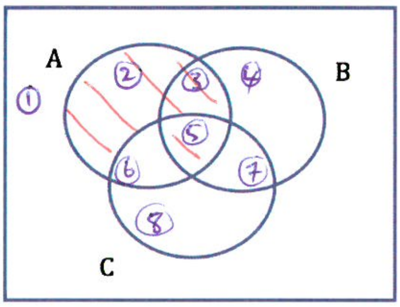
✓ identifies, using set notation

b) Shade $(B \cup \bar{C}) \cap A$

$(B \cup \bar{C})$ areas $= \{3, 4, 5, 7, 1, 2\}$

A areas $= \{2, 3, 5, 6\}$

$(B \cup \bar{C}) \cap A = \{3, 5, 2\}$



✓ shades correctly

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Question 4 **[2, 3 = 5 marks]**

Given $P(A) = 0.8 - x$, $P(A \cap B) = 0.1 + x$ and $P(B) = 0.4$,

a) if $P(\bar{A}) = 0.6$, determine the value x.

$P(\bar{A}) = 1 - P(A)$

$0.6 = 1 - (0.8 - x)$ ✓ substitutes

$0.6 = 1 - 0.8 + x$

$\underline{x = 0.4}$ ✓ solves

* CHECK use of brackets around binomial terms

$x = -0.1$ (1 FT)

b) determine the value of x if the events A and B are independent.

$P(A \cap B) = P(A) \times P(B)$

$0.1 + x = (0.8 - x)\,0.4$ ✓ substitutes

$0.1 + x = 0.32 - 0.4x$

$1.4x = 0.22$ ✓ simplifies

$x = \frac{22}{140}$ or $\underline{\frac{11}{70}}$ ✓ answers in fractional form

$x = 0.35$, $x = \frac{31}{140}$, $x = 0.11$ } 1 FT

0.06

Question 5 **[4, 3 = 7 marks]**

a) Use the two-way table and the information below to determine the value of $n(U)$.

$n(A) = 32$ $n(B) = 24$ $n(\overline{A \cup B}) = 7$ $n(U) = 8 \times n(A \cap B)$

	B	$\bar{B}$	TOTAL
A	x	$32 - x$	32
$\bar{A}$	$24 - x$	7	$8x - 32$
TOTAL	24	$8x - 24$	$8x$

✓ $A, B, \overline{A \cup B}$ shown

✓ { uses algebra OR completes table numerically

✓ solves $n(A \cap B)$

Solve eq $32 - x + 7 = 8x - 24$

$9x = 63$

$\underline{x = 7}$

$n(U) = 8 \times 7$

$\underline{= 56}$

✓ state $n(U) = 56$

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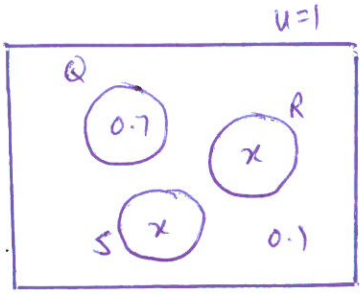
b) Construct a Venn diagram showing three mutually exclusive events Q, R and S such that:

- events R and S are equally likely to occur;
- the probability of event Q not occurring is 0.3; and,
- $P(\overline{Q \cup R \cup S}) = 0.1$.



$x = 0.1$

✓ constructs three separate "circles"

✓ solves for x, $P(R) = P(S) = 0.1$

✓ labels areas: 0.7, 0.1 $\left\{ \begin{matrix} 0.1, 0.1 \\ x, x \end{matrix} \right.$

Full marks if intersections labelled as having 0 elements

Question 6. **[2, 2, 3, 1 = 8 marks]**

a) Expand $(a + b)^4$.

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

✓ lists five terms $a^4, a^3b, a^2b^2, ab^3, b^4$

✓ determines coefficients

b) Using the properties of Pascal's triangle, determine the sum of the coefficients of $(a + b)^2 + (a + b)^3 + (a + b)^4$.

$= 2^2 + 2^3 + 2^4$

$= 4 + 8 + 16$

$= \underline{\underline{28}}$

✓ uses 2^n

✓ evaluates

c) Without showing the expansion, identify the coefficient of p^3 in the expansion of $(1 - 3p)^5$.

$\binom{5}{2}(1)^2(-3)^3$

$= 10 \times (-27)$

$= \underline{\underline{-270}}$

✓ identifies $\binom{5}{2}$ or $\binom{5}{3}$

✓ includes $(-3)^3$

✓ evaluates

Answer -30 with working 2 FT

d) Briefly explain why the sum of the coefficients in $(1 - 3p)^5$ will not equal 2^5, as implied by the fifth row of Pascal's triangle.

Consider generic binomial $(a+b)^n$

In this case b is $(-3p)$, with a coefficient different to 1

✓ provides meaningful explanation related to -3

End of Calculator-Free Section

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Year 11 Mathematics Methods
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Question 7. **[2, 1, 2= 5 marks]**

Two friends, Penn and Teller, are playing a guessing game. Penn is to choose two different letters of the alphabet and write them down on a piece of card. The order in which Penn selects the letters is **not** important.

How many two-letter combinations can Penn choose:

a) if there are no restrictions?

$^{26}C_2$ or $\binom{26}{2} = \underline{\underline{325}}$

✓ shows use $\binom{n}{r}$

✓ calculates

b) if he chooses one vowel and one consonant?

$\binom{5}{1}\binom{21}{1} = \underline{\underline{105}}$

✓ calculates

c) if he chooses either both as consonants or both as vowels?

$\binom{5}{2} + \binom{21}{2}$ ✓ adds scenarios

$= 10 + 210$

$= 220$ ✓ calculates

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Question 8. **[3, 1, 3, 3 = 10 marks]**

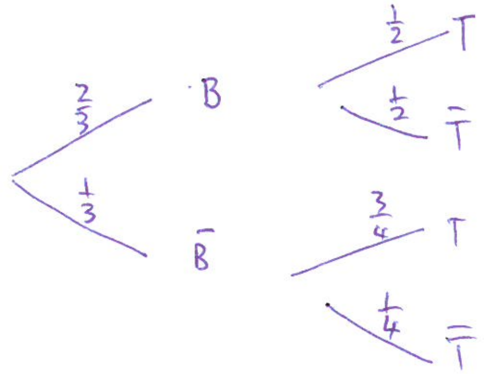
On the morning of the 26th February, 360 Year 11 students were surveyed about whether they ate breakfast at home in the morning and whether they took public transport to school.

Two-thirds of the Year 11 students ate breakfast at home before coming to school, and of these, half of the students caught public transport. 25% of the students who did not eat breakfast at home before coming to school did not catch public transport.

Let B be the event that a student eats breakfast at home and T be the event that a student takes public transport to school.

a) Construct a weighted tree diagram to represent this situation, showing the complete sample space.

Event One Event Two Sample Space



BT

$B\bar{T}$

$\bar{B}T$

$\bar{B}\bar{T}$

✓ constructs tree

✓ weights branches correctly

✓ lists sample space

b) Determine the probability that a randomly selected Year 11 student from this survey group ate breakfast at home and caught public transport.

$P(B \cap T) = \frac{2}{3} \times \frac{1}{2}$

$= \frac{2}{6}$ or $\frac{1}{3}$ ✓ calculates

c) Determine the total number of students that caught public transport.

$P(T) = P(BT) + P(\bar{B}T)$ ✓ adds

$= \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{4}$

$= \frac{2}{6} + \frac{3}{12} = \frac{7}{12}$ ✓ calculates P(T)

$n(T) = \frac{7}{12} \times 360$

$= 210$ ✓ converts to number

d) Using the data from this survey, estimate the number of Year 11 students out of a larger population of 12 500 that would eat breakfast at home, but not catch public transport. Answer to the nearest 10.

$P(B \cap T') = \frac{2}{3} \times \frac{1}{2}$

$= \frac{1}{3}$ ✓ calculates $P(B\bar{T})$

Estimate $= \frac{1}{3} \times 12500$ ✓ multiplies

$= 4170$ to the nearest 10

✓ answers to nearest 10

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Question 9. **[2, 1, 1, 2 = 6 marks]**

A corner deli recorded its lunchtime sales for a week. During this time, it served a total of 621 customers. 233 customers bought a burger and a cup of hot chips. An additional 189 customers bought just a cup of hot chips. 94 customers bought items other than burgers or chips.

a) Complete the two-way table below.

	B	$\bar{B}$	TOTAL
C	233	189	422
$\bar{C}$	105	94	199
TOTAL	338	283	621

✓ populates 189
✓ completes table

b) Determine the probability (to 4 decimal places):

(i) that a customer bought a burger;

$P(B) = \frac{338}{621}$

≈ 0.5443 (to 4 dp) ✓ calculates

(ii) that a customer bought a burger, given that they bought chips;

$P(B|C) = \frac{233}{422}$

≈ 0.5521 (to 4 dp) ✓ calculates

-1 not 4 dp

c) Hence, comment on whether there is sufficient evidence to indicate that a customer buying a burger is independent of the customer buying chips.

Since P(B) and P(B|C) are within ± 5% we can infer independence

✓ meaningful comparison, ± 5%

✓ concludes correctly

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Question 10. **[6 marks]**

A committee of five current Year 11 students is to be established to plan next year's Year 12 ball. Seven boys and eight girls put their names forward to be on the committee.

Given that the committee must have at least one boy, determine the probability that it has more boys than girls.

Event A : committee has at least one boy

$$n(A) = \binom{15}{5} - \underbrace{\binom{7}{0}\binom{8}{5}}_{\text{no boys}}$$

✓ subtracts no boys

$= 3003 - 56$

$= 2947$ ✓ calculates $n(A)$

Event B : committee has more boys than girls

✓ adds three scenarios

$$n(B) = \binom{7}{3}\binom{8}{2} + \binom{7}{4}\binom{8}{1} + \binom{7}{5}\binom{8}{0}$$

✓ correct $\binom{n}{r}$

$= \underset{980}{35 \times 28} + \underset{280}{35 \times 8} + 21$

$= 1281$ ✓ calculates $n(B)$

$P(B|A) = \dfrac{1281}{2947}$ $\dfrac{183}{421}$

≈ 0.4347 (to 4 dp) ✓ calculates $P(B|A)$

* Definition of events is not required.

End of Calculator Assumed Section

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